Similarity Search

CSE545 - Spring 2022

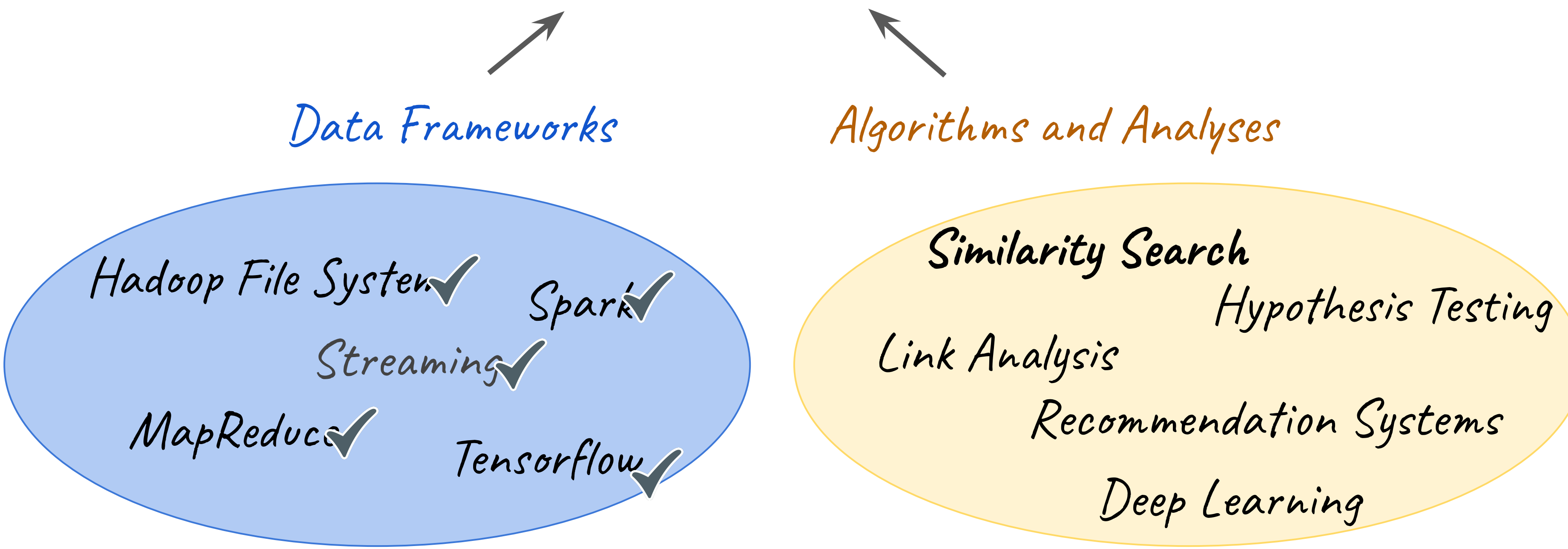
Stony Brook University

H. Andrew Schwartz

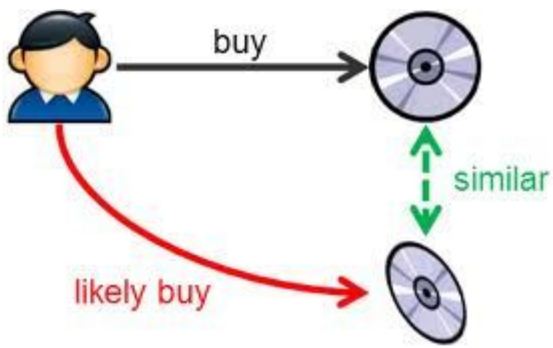
$A \cap B$

Big Data Analytics, The Class

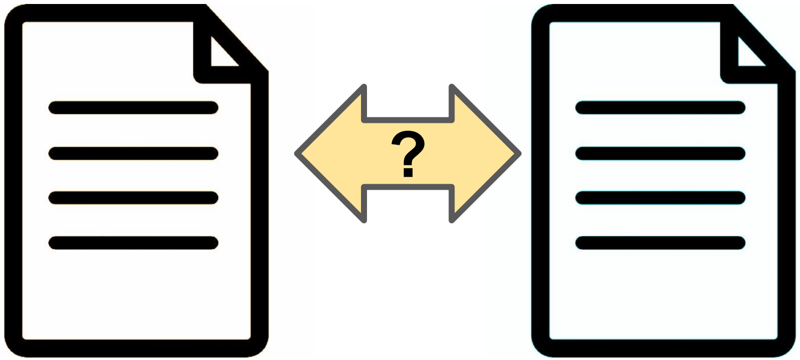
Goal: Generalizations
A *model* or *summarization* of the data.

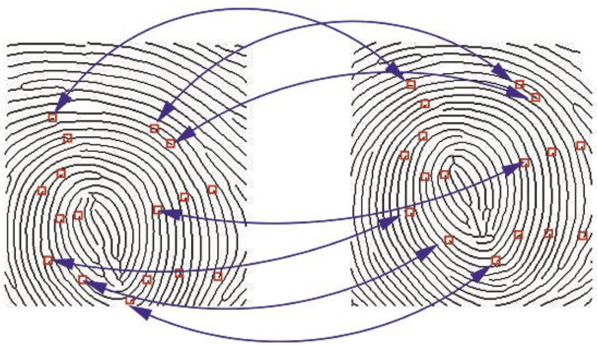


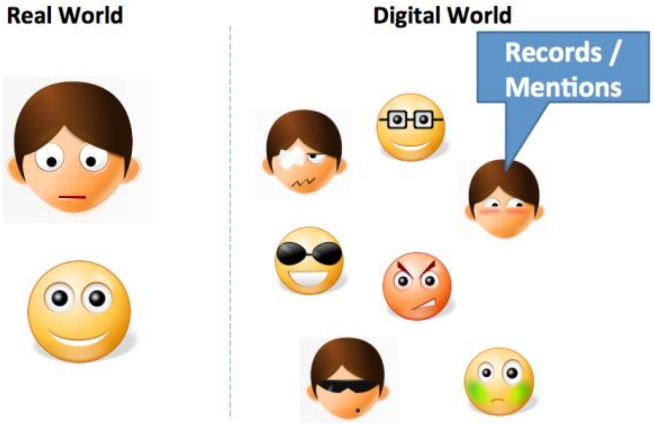
Finding Similar Items



(http://blog.soton.ac.uk/hive/2012/05/10/recommendation-system-of-hive/)







(http://www.datacommunitydc.org/blog/2013/08/entity-resolution-for-big-data)

Finding Similar Items: Topics

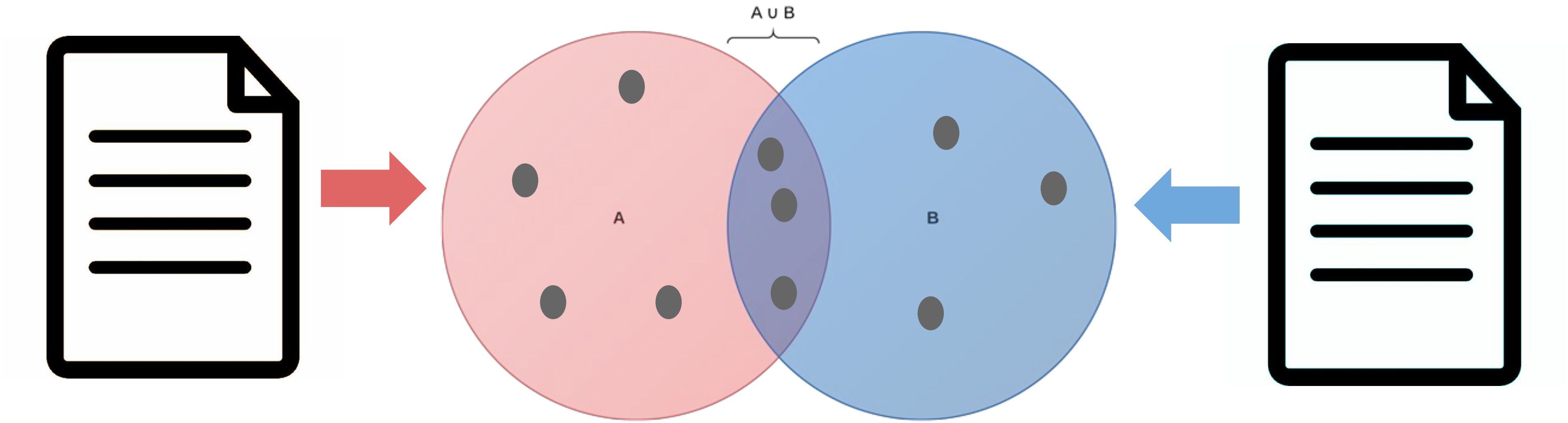
- Shingling
- Minhashing
- Locality-sensitive hashing
- Distance Metrics

Document Similarity

Challenge: How to represent the document in a way that can be efficiently encoded and compared?

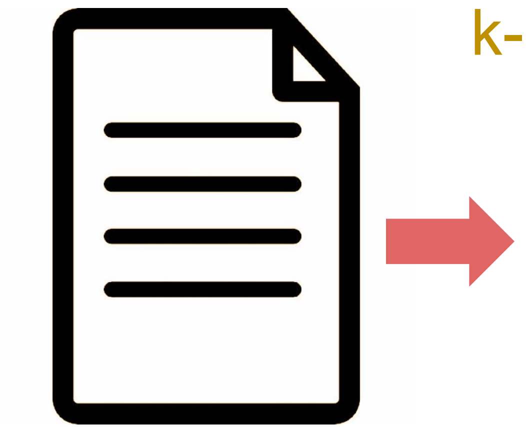
Shingles

Goal: Convert documents to sets



Shingles

Goal: Convert documents to sets

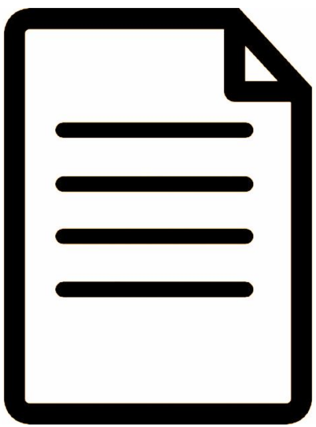


k-shingles (aka "character n-grams")
- sequence of *k* characters

E.g. k=2 doc="abcdabd"
singles(doc, 2) = {ab, bc, cd, da, bd}

Shingles

Goal: Convert documents to sets



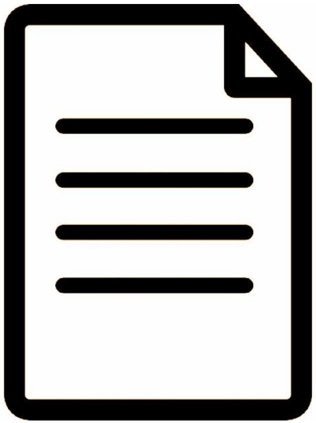
k-shingles (aka “character n-grams”)
- sequence of *k* characters

E.g. *k*=2 doc=”abcdabd”
singles(doc, 2) = {ab, bc, cd, da, bd}

- Similar documents have many common shingles
- Changing words or order has minimal effect.
- In practice use 5 < k < 10

Shingles

Goal: Convert documents to sets



Large enough that any given shingle appearing a document is highly unlikely
(e.g. < .1% chance)

Can hash large shingles to smaller
(e.g. 9-shingles into 4 bytes)

Can also use words (aka n-grams).

- Similar documents have many common shingles
- Changing words or order has minimal effect.
- **In practice use 5 < k < 10**

Shingles

Problem: Even if hashing, sets of shingles are large
(e.g. 4 bytes => 4x the size of the document).

Minhashing

Goal: Convert sets to shorter ids, signatures

Minhashing

Goal: Convert sets to shorter ids, "signatures"

Characteristic Matrix, X:

Element	S_1	S_2	S_3	S_4
a	1	0	0	1
b	0	0	1	0
c	0	1	0	1
d	1	0	1	1
e	0	0	1	0

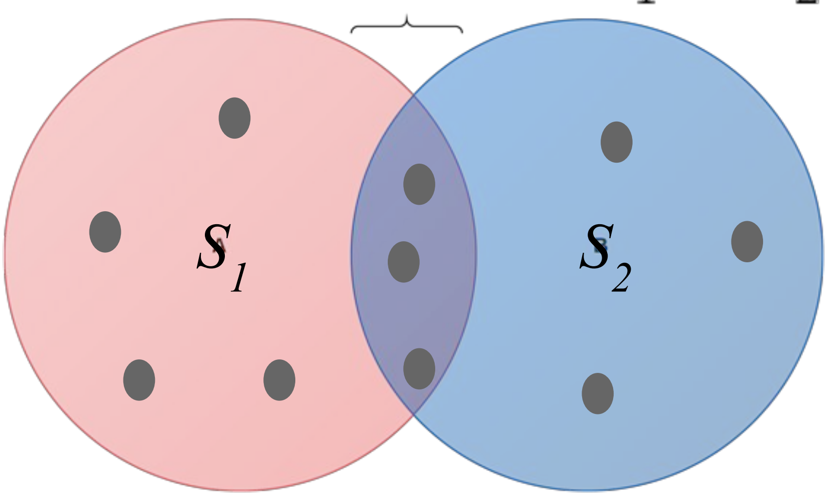
….

(Leskovec at al., 2014; http://www.mmds.org/)

often very sparse! (lots of zeros)

Jaccard Similarity:

$$sim(S_1, S_2) = \frac{S_1 \cap S_2}{S_1 \cup S_2}$$



Minhashing

Characteristic Matrix:

	S_1	S_2
ab	1	1
bc	0	1
de	1	0
ah	1	1
ha	0	0
ed	1	1
ca	0	1

Jaccard Similarity:

$$sim(S_1, S_2) = \frac{S_1 \cap S_2}{S_1 \cup S_2}$$

Minhashing

Characteristic Matrix:

	S_1	S_2	
ab	1	1	* *
bc	0	1	*
de	1	0	*
ah	1	1	**
ha	0	0	
ed	1	1	**
ca	0	1	*

Jaccard Similarity:

$$sim(S_1, S_2) = \frac{S_1 \cap S_2}{S_1 \cup S_2}$$

Minhashing

Characteristic Matrix:

	S_1	S_2	
ab	1	1	* *
bc	0	1	*
de	1	0	*
ah	1	1	**
ha	0	0	
ed	1	1	**
ca	0	1	*

Jaccard Similarity:

$$sim(S_1, S_2) = \frac{S_1 \cap S_2}{S_1 \cup S_2}$$

$sim(S_1, S_2) = 3 / 6$

both have / # at least one has

Minhashing

Problem: Even if hashing shingle contents,
sets of shingles are large

e.g. 4 byte integer per shingle: assume all unique shingles,
=> 4x the size of the document

(since there are as many shingles as characters and 1byte per char).

Minhashing

Goal: Convert sets to shorter ids, "signatures"

Characteristic Matrix: X

	S_1	S_2	S_3	S_4
ab	1	0	1	0
bc	1	0	0	1
de	0	1	0	1
ah	0	1	0	1
ha	0	1	0	1
ed	1	0	1	0
ca	1	0	1	0

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Goal: Convert sets to shorter ids, "signatures"

Characteristic Matrix: X

	S_1	S_2	S_3	S_4
ab	1	0	1	0
bc	1	0	0	1
de	0	1	0	1
ah	0	1	0	1
ha	0	1	0	1
ed	1	0	1	0
ca	1	0	1	0

(Leskovec at al., 2014; http://www.mmds.org/)

Approximate Approach:

1) Instead of keeping whole characteristic matrix, just keep first row where 1 is encountered.

2) Shuffle and repeat to get a "signature" for each set.

Minhashing

Goal: Convert sets to shorter ids, "signatures"

Characteristic Matrix: X

1 3 1 2

	S_1	S_2	S_3	S_4
ab	1	0	1	0
bc	1	0	0	1
de	0	1	0	1
ah	0	1	0	1
ha	0	1	0	1
ed	1	0	1	0
ca	1	0	1	0

(Leskovec at al., 2014; http://www.mmds.org/)

Approximate Approach:

1) Instead of keeping whole characteristic matrix, just keep first row where 1 is encountered.

2) Shuffle and repeat to get a "signature" for each set.

Minhashing

Goal: Convert sets to shorter ids, "signatures"

Characteristic Matrix: X

	1	3	1	2
	S_1	S_2	S_3	S_4
ab	1	0	1	0
bc	1	0	0	1
de	0	1	0	1
ah	0	1	0	1
ha	0	1	0	1
ed	1	0	1	0
ca	1	0	1	0

(Leskovec at al., 2014; http://www.mmds.org/)

Approximate Approach:

1) Instead of keeping whole characteristic matrix, just keep first row where 1 is encountered.

2) Shuffle and repeat to get a "signature".

	2	1	2	1
	S_1	S_2	S_3	S_4
ah	0	1	0	1
ca	1	0	1	0
ed	1	0	1	0
de	0	1	0	1
ab	1	0	1	0
bc	1	0	0	1

...

Minhashing

Goal: Convert sets to shorter ids, "signatures"

Characteristic Matrix: X

1 3 1 2

	S_1	S_2	S_3	S_4
ab	1	0	1	0
bc	1	0	0	1
de	0	1	0	1
ah	0	1	0	1
ha	0	1	0	1
ed	1	0	1	0
ca	1	0	1	0

(Leskovec at al., 2014; http://www.mmds.org/)

Approximate Approach:

1) Instead of keeping whole characteristic matrix, just keep first row where 1 is encountered.

2) Shuffle and repeat to get a "signature".

2 1 2 1

	S_1	S_2	S_3	S_4
ah	0	1	0	1
ca	1	0	1	0
ed	1	0	1	0
de	0	1	0	1
ab	1	0	1	0
bc	1	0	0	1

...

signatures

S_1	S_2	S_3	S_4
1	3	1	2
2	1	2	1
...	...	...	...

Minhashing

Goal: Convert sets to shorter ids, “signatures”

Characteristic Matrix: X

	S_1	S_2	S_3	S_4
ab	1	0	1	0
bc	1	0	0	1
de	0	1	0	1
ah	0	1	0	1
ha	0	1	0	1
ed	1	0	1	0
ca	1	0	1	0

(Leskovec at al., 2014; http://www.mmds.org/)

Approximate Approach:

1) Instead of keeping whole characteristic matrix, just keep first row where 1 is encountered.

2) Shuffle and repeat to get a “signature” for each set.

Idea: We don’t need to actually shuffle. We can just permute row ids.

Minhashing

Characteristic Matrix:

	S_1	S_2	S_3	S_4
ab	1	0	1	0
bc	1	0	0	1
de	0	1	0	1
ah	0	1	0	1
ha	0	1	0	1
ed	1	0	1	0
ca	1	0	1	0

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to first row where set appears.

Minhashing

Characteristic Matrix:

	S_1	S_2	S_3	S_4
ab	1	0	1	0
bc	1	0	0	1
de	0	1	0	1
ah	0	1	0	1
ha	0	1	0	1
ed	1	0	1	0
ca	1	0	1	0

permuted order
1 ha
2 ed
3 ab
4 bc
5 ca
6 ah
7 de

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to first row where set appears.

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Characteristic Matrix:

		S_1	S_2	S_3	S_4
3	ab	1	0	1	0
4	bc	1	0	0	1
7	de	0	1	0	1
6	ah	0	1	0	1
1	ha	0	1	0	1
2	ed	1	0	1	0
5	ca	1	0	1	0

permuted order
1 ha
2 ed
3 ab
4 bc
5 ca
6 ah
7 de

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to first row where set appears.

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to first row where set appears.

Characteristic Matrix:

		S_1	S_2	S_3	S_4
3	ab	1	0	1	0
4	bc	1	0	0	1
7	de	0	1	0	1
6	ah	0	1	0	1
1	ha	0	1	0	1
2	ed	1	0	1	0
5	ca	1	0	1	0

permuted order
1 ha
2 ed
3 ab
4 bc
5 ca
6 ah
7 de

$h(S_1)$ = ed #permuted row 2

$h(S_2)$ = ha #permuted row 1

$h(S_3)$ =

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Characteristic Matrix:

		S_1	S_2	S_3	S_4
3	ab	1	0	1	0
4	bc	1	0	0	1
7	de	0	1	0	1
6	ah	0	1	0	1
1	ha	0	1	0	1
2	ed	1	0	1	0
5	ca	1	0	1	0

permuted order
1 ha
2 ed
3 ab
4 bc
5 ca
6 ah
7 de

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to first row where set appears.

$h(S_1)$ = ed #permuted row 2
$h(S_2)$ = ha #permuted row 1
$h(S_3)$ = ed #permuted row 2
$h(S_4)$ =

Minhashing

Characteristic Matrix:

		S_1	S_2	S_3	S_4
3	ab	1	0	1	0
4	bc	1	0	0	1
7	de	0	1	0	1
6	ah	0	1	0	1
1	ha	0	1	0	1
2	ed	1	0	1	0
5	ca	1	0	1	0

permuted order
1 ha
2 ed
3 ab
4 bc
5 ca
6 ah
7 de

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to first row where set appears.

$h(S_1)$ = ed #permuted row 2
$h(S_2)$ = ha #permuted row 1
$h(S_3)$ = ed #permuted row 2
$h(S_4)$ = ha #permuted row 1

Minhashing

Characteristic Matrix:

		S_1	S_2	S_3	S_4
3	ab	1	0	1	0
4	bc	1	0	0	1
7	de	0	1	0	1
6	ah	0	1	0	1
1	ha	0	1	0	1
2	ed	1	0	1	0
5	ca	1	0	1	0

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to rows.

Signature matrix: *M*

- Record first row where each set had a 1 in the given permutation

	S_1	S_2	S_3	S_4
h_1	2	1	2	1

$h_1(S_1)$ = ed #permuted row 2
$h_1(S_2)$ = ha #permuted row 1
$h_1(S_3)$ = ed #permuted row 2
$h_1(S_4)$ = ha #permuted row 1

Minhashing

Characteristic Matrix:

		S_1	S_2	S_3	S_4
3	ab	1	0	1	0
4	bc	1	0	0	1
7	de	0	1	0	1
6	ah	0	1	0	1
1	ha	0	1	0	1
2	ed	1	0	1	0
5	ca	1	0	1	0

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to rows.

Signature matrix: M

- Record first row where each set had a 1 in the given permutation

	S_1	S_2	S_3	S_4
h_1	2	1	2	1

$h_1(S_1)$ = ed #permuted row 2

$h_1(S_2)$ = ha #permuted row 1

$h_1(S_3)$ = ed #permuted row

Minhashing

Characteristic Matrix:

		S_1	S_2	S_3	S_4
3	ab	1	0	1	0
4	bc	1	0	0	1
7	de	0	1	0	1
6	ah	0	1	0	1
1	ha	0	1	0	1
2	ed	1	0	1	0
5	ca	1	0	1	0

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to rows.

Signature matrix: M

- Record first row where each set had a 1 in the given permutation

	S_1	S_2	S_3	S_4
h_1	2	1	2	1

$h_1(S_1)$ = ed #permuted row

2

$h_1(S_2)$ = ha #permuted row

1

$h_1(S_3)$ = ed #permuted row

Minhashing

Characteristic Matrix:

			S_1	S_2	S_3	S_4
4	3	ab	1	0	1	0
2	4	bc	1	0	0	1
1	7	de	0	1	0	1
3	6	ah	0	1	0	1
6	1	ha	0	1	0	1
7	2	ed	1	0	1	0
5	5	ca	1	0	1	0

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to rows.

Signature matrix: *M*

- Record first row where each set had a 1 in the given permutation

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2				

Minhashing

Characteristic Matrix:

			S_1	S_2	S_3	S_4
4	3	ab	1	0	1	0
2	4	bc	1	0	0	1
1	7	de	0	1	0	1
3	6	ah	0	1	0	1
6	1	ha	0	1	0	1
7	2	ed	1	0	1	0
5	5	ca	1	0	1	0

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to rows.

Signature matrix: *M*

- Record first row where each set had a 1 in the given permutation

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2	2	1	4	1

Minhashing

Characteristic Matrix:

				S_1	S_2	S_3	S_4
1	4	3	ab	1	0	1	0
3	2	4	bc	1	0	0	1
7	1	7	de	0	1	0	1
6	3	6	ah	0	1	0	1
2	6	1	ha	0	1	0	1
5	7	2	ed	1	0	1	0
4	5	5	ca	1	0	1	0

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to rows.

Signature matrix: M

- Record first row where each set had a 1 in the given permutation

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2	2	1	4	1
h_3				

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Characteristic Matrix:

				S_1	S_2	S_3	S_4
1	4	3	ab	1	0	1	0
3	2	4	bc	1	0	0	1
7	1	7	de	0	1	0	1
6	3	6	ah	0	1	0	1
2	6	1	ha	0	1	0	1
5	7	2	ed	1	0	1	0
4	5	5	ca	1	0	1	0

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to rows.

Signature matrix: *M*

- Record first row where each set had a 1 in the given permutation

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2	2	1	4	1
h_3	1	2	1	2

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Characteristic Matrix:

				S_1	S_2	S_3	S_4
1	4	3	ab	1	0	1	0
3	2	4	bc	1	0	0	1
7	1	7	de	0	1	0	1
6	3	6	ah	0	1	0	1
2	6	1	ha	0	1	0	1
5	7	2	ed	1	0	1	0
4	5	5	ca	1	0	1	0

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to rows.

Signature matrix: *M*

- Record first row where each set had a 1 in the given permutation

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2	2	1	4	1
h_3	1	2	1	2
...				
...				

Minhashing

Property of signature matrix:
The probability for any h_i (i.e. any row), that $h_i(S_1) = h_i(S_2)$ is the same as Sim(S_1, S_2)

Characteristic Matrix:

				S_1	S_2	S_3	S_4
1	4	3	ab	1	0	1	0
3	2	4	bc	1	0	0	1
7	1	7	de	0	1	0	1
6	3	6	ah	0	1	0	1
2	6	1	ha	0	1	0	1
5	7	2	ed	1	0	1	0
4	5	5	ca	1	0	1	0

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2	2	1	4	1
h_3	1	2	1	2
...				
...				

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Property of signature matrix:
The probability for any h_i (i.e. any row), that $h_i(S_1) = h_i(S_2)$ is the same as Sim(S_1, S_2)

Thus, similarity of signatures S_1, S_2 is the fraction of minhash functions (i.e. rows) in which they agree.

Characteristic Matrix:

				S_1	S_2	S_3	S_4
1	4	3	ab	1	0	1	0
3	2	4	bc	1	0	0	1
7	1	7	de	0	1	0	1
6	3	6	ah	0	1	0	1
2	6	1	ha	0	1	0	1
5	7	2	ed	1	0	1	0
4	5	5	ca	1	0	1	0

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2	2	1	4	1
h_3	1	2	1	2
...				
...				

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Property of signature matrix:
The probability for any h_i (i.e. any row), that $h_i(S_1) = h_i(S_2)$ is the same as Sim(S_1, S_2)

Thus, similarity of signatures S_1, S_2 is the fraction of minhash functions (i.e. rows) in which they agree.

Characteristic Matrix:

				S_1	S_2	S_3	S_4
1	4	3	ab	1	0		0
3	2	4					
7							
6	3	6	ah				
2	6	1	ha	0	1	0	1
5	7	2	ed	1	0	1	0
4	5	5	ca	1	0	1	0

Estimate with a random sample of permutations (i.e. ~100)

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2	2	1	4	1
h_3	1	2	1	2
...				
...				

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Characteristic Matrix:

Property of signature matrix:
The probability for any h_i (i.e. any row), that $h_i(S_1) = h_i(S_2)$ is the same as Sim(S_1, S_2)

Thus, similarity of signatures S_1, S_2 is the fraction of minhash functions (i.e. rows) in which they agree.

				S_1	S_2	S_3	S_4
1	4	3	ab	1	0	[illegible]	0
3	2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6	3	6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2	6	1	ha	0	1	0	1
5	7	2	ed	1	0	1	0
4	5	5	ca	1	0	1	0

Estimate with a random sample of permutations (i.e. ~100)

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2	2	1	4	1
h_3	1	2	1	2

Estimated Sim(S_1, S_3) = agree / all = 2/3

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Property of signature matrix:
The probability for any h_i (i.e. any row), that $h_i(S_1) = h_i(S_2)$ is the same as Sim(S_1, S_2)

Thus, similarity of signatures S_1, S_2 is the fraction of minhash functions (i.e. rows) in which they agree.

Characteristic Matrix:

				S_1	S_2	S_3	S_4
1	4	3	ab	1	0	1	0
3	2	4	bc	1	0	0	1
7	1	7	de	0	1	0	1
6	3	6	ah	0	1	0	1
2	6	1	ha	0	1	0	1
5	7	2	ed	1	0	1	0
4	5	5	ca	1	0	1	0

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2	2	1	4	1
h_3	1	2	1	2

Estimated Sim(S_1, S_3) = agree / all = 2/3

Real Sim(S_1, S_3) = Type a / (a + b + c) = 3/4

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Characteristic Matrix:

				S_1	S_2	S_3	S_4
1	4	3	ab	1	0	1	0
3	2	4	bc	1	0	0	1
7	1	7	de	0	1	0	1
6	3	6	ah	0	1	0	1
2	6	1	ha	0	1	0	1
5	7	2	ed	1	0	1	0
4	5	5	ca	1	0	1	0

Property of signature matrix:
The probability for any h_i (i.e. any row), that $h_i(S_1) = h_i(S_2)$ is the same as Sim(S_1, S_2)

Thus, similarity of signatures S_1, S_2 is the fraction of minhash functions (i.e. rows) in which they agree.

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2	2	1	4	1
h_3	1	2	1	2

Estimated Sim(S_1, S_3) = agree / all = 2/3

Real Sim(S_1, S_3) = Type a / (a + b + c) = 3/4

Try Sim(S_2, S_4) and Sim(S_1, S_2)

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Error Bound?

Characteristic Matrix:

				S_1	S_2	S_3	S_4
1	4	3	ab	1	0	1	0
3	2	4	bc	1	0	0	1
7	1	7	de	0	1	0	1
6	3	6	ah	0	1	0	1
2	6	1	ha	0	1	0	1
5	7	2	ed	1	0	1	0
4	5	5	ca	1	0	1	0

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2	2	1	4	1
h_3	1	2	1	2

Estimated Sim(S_1, S_3) = agree / all = 2/3

Real Sim(S_1, S_3) = Type a / (a + b + c) = 3/4

Try Sim(S_2, S_4) and Sim(S_1, S_2)

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Characteristic Matrix:

				S_1	S_2	S_3	S_4
1	4	3	ab	1	0	1	0
3	2	4	bc	1	0	0	1
7	1	7	de	0	1	0	1
6	3	6	ah	0	1	0	1
2	6	1	ha	0	1	0	1
5	7	2	ed	1	0	1	0
4	5	5	ca	1	0	1	0

(Leskovec at al., 2014; http://www.mmds.org/)

Error Bound?

Expect error: **$O(1/\sqrt{k})$** (k hashes)

Why? Each row is a random observation of 1 or 0 (match or not) with *P(match=1) = Sim(S1, S2).*

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2	2	1	4	1
h_3	1	2	1	2

Estimated Sim(S_1, S_3) = agree / all = 2/3

Real Sim(S_1, S_3) = Type a / (a + b + c) = 3/4

Try Sim(S_2, S_4) and Sim(S_1, S_2)

Minhashing

Characteristic Matrix:

				S_1	S_2	S_3	S_4
1	4	3	ab	1	0	1	0
3	2	4	bc	1	0	0	1
7	1	7	de	0	1	0	1
6	3	6	ah	0	1	0	1
2	6	1	ha	0	1	0	1
5	7	2	ed	1	0	1	0
4	5	5	ca	1	0	1	0

(Leskovec at al., 2014; http://www.mmds.org/)

Error Bound?

Expect error: **O(*1*/√*k*)** (k hashes)

Why? Each row is a random observation of 1 or 0 (match or not) with *P(match=1) = Sim(S1, S2).*

N = *k* observations

Standard deviation(*std*)? < 1 (worst case is 0.5)

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2	2	1	4	1
h_3	1	2	1	2

Estimated Sim(S_1, S_3) = agree / all = 2/3

Real Sim(S_1, S_3) = Type a / (a + b + c) = 3/4

Try Sim(S_2, S_4) and Sim(S_1, S_2)

Minhashing

Characteristic Matrix:

				S_1	S_2	S_3	S_4
1	4	3	ab	1	0	1	0
3	2	4	bc	1	0	0	1
7	1	7	de	0	1	0	1
6	3	6	ah	0	1	0	1
2	6	1	ha	0	1	0	1
5	7	2	ed	1	0	1	0
4	5	5	ca	1	0	1	0

(Leskovec at al., 2014; http://www.mmds.org/)

Error Bound?

Expect error: $\mathbf{O(1/\sqrt{k})}$ (k hashes)

Why? Each row is a random observation of 1 or 0 (match or not) with *P(match=1) = Sim(S1, S2).*

N = *k* observations

Standard deviation(*std*)? < 1 (worst case is 0.5)

Standard Error of Mean = *std*/√N

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2	2	1	4	1
h_3	1	2	1	2

Estimated Sim(S_1, S_3) = agree / all = 2/3

Real Sim(S_1, S_3) = Type a / (a + b + c) = 3/4

Try Sim(S_2, S_4) and Sim(S_1, S_2)

Minhashing

In Practice

Problem:

- Can’t reasonably do permutations (huge space)
- Can’t randomly grab rows according to an order (random disk seeks = slow!)

Minhashing

In Practice

Problem:

- Can’t reasonably do permutations (huge space)
- Can’t randomly grab rows according to an order (random disk seeks = slow!)

Solution: Use “random” hash functions.

- Setup:
  - Pick ~100 hash functions, hashes
  - Store M[i][s] = a potential minimum $h_i(r)$
    #initialized to infinity (num hashs x num sets)

Minhashing

Solution: Use "random" hash functions.

Setup:

```
hashes = [getHfunc(i) for i in rand(1, num=100)]
                                    #100 hash functions, seeded random
for i in hashes: for s in sets:
    Sig[i][s] = np.inf #represents a potential minimum h_i(r) ; initially infinity
```

Minhashing

Solution: Use "random" hash functions.

Setup:

```
hashes = [getHfunc(i) for i in rand(1, num=100)]
                              #100 hash functions, seeded random
for i in hashes: for s in sets:
    Sig[i][s] = np.inf #represents a potential minimum h_i(r) ; initially infinity
```

Algorithm ("efficient minhashing"):

```
for r in rows of cm: #cm is characteristic matrix
  compute h_i(r) for all i in hashes #precompute 100 values
  for each set s in sets: #columns of cm
    if cm[r][s] == 1:
      for i in hashes: #check which hash produces smallest value
        if h_i(r) < Sig[i][s]: Sig[i][s] = h_i(r)
```

Minhashing

Solution: Use "random" hash functions.

Setup:

hashes = [getHfunc(i) for i in rand(1, num=100)]

#100 hash functions, seeded random

for i in hashes: for s in sets:

  Sig[i][s] = np.inf *#represents a potential minimum $h_i(r)$; initially infinity*

Algorithm ("efficient minhashing") without charact matrix:

```
for feat in shins: #shins is all unique shingles
 compute h_i(feat) for all i in hashes #precompute 100 values
 for each set s in sets: #sets is list of shingle sets
   if feat in s:
     for i in hashes: #check which hash produces smallest value
       if h_i(feat) < Sig[i][s_id]: Sig[i][s_id] = h_i(feat)
```

Minhashing

Problem: Even if hashing, sets of shingles are large (e.g. 4 bytes => 4x the size of the document).

Minhashing

Problem: Even if hashing, sets of shingles are large (e.g. 4 bytes => 4x the size of the document).

New Problem: Even if the size of signatures are small, it can be computationally expensive to find similar pairs.

E.g. 1m documents; 1,000,000 choose 2 = 500,000,000,000 pairs!

Minhashing

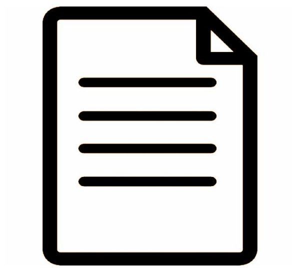
Problem: Even if hashing, sets of shingles are large (e.g. 4 bytes => 4x the size of the document).

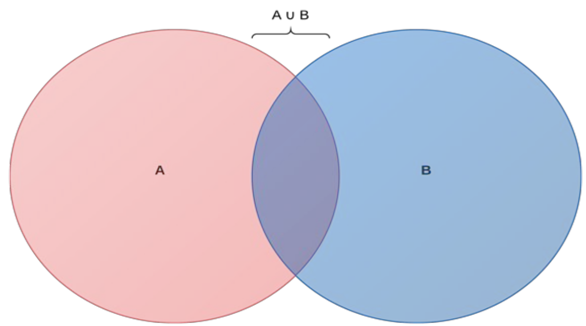
New Problem: Even if the size of signatures are small, it can be computationally expensive to find similar pairs.

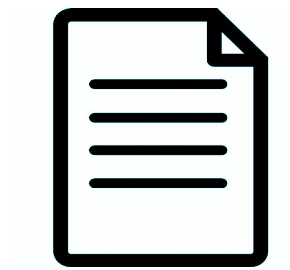
E.g. 1m documents; 1,000,000 choose 2 = 500,000,000,000 pairs!

(1m documents isn't even "big data")

Document Similarity







Duplicate web pages (useful for ranking

Plagiarism

Cluster News Articles

Anything similar to documents: movie/music/art tastes, product characteristics

COVID-19 Report matching

Locality-Sensitive Hashing

Goal: find pairs of minhashes ***likely*** to be similar (in order to then test more precisely for similarity).

Candidate pairs: pairs of elements to be evaluated for similarity.

Locality-Sensitive Hashing

Goal: find pairs of minhashes ***likely*** to be similar (in order to then test more precisely for similarity).

Candidate pairs: pairs of elements to be evaluated for similarity.

If we wanted the similarity for all pairs of documents, could anything be done?

Locality-Sensitive Hashing

Goal: find pairs of minhashes likely to be similar (in order to then test more precisely for similarity).

Candidate pairs: pairs of elements to be evaluated for similarity.

Approach: Hash multiple times over subsets of data: similar items are likely in the same bucket once.

Locality-Sensitive Hashing

Goal: find pairs of minhashes likely to be similar (in order to then test more precisely for similarity).

Candidate pairs: pairs of elements to be evaluated for similarity.

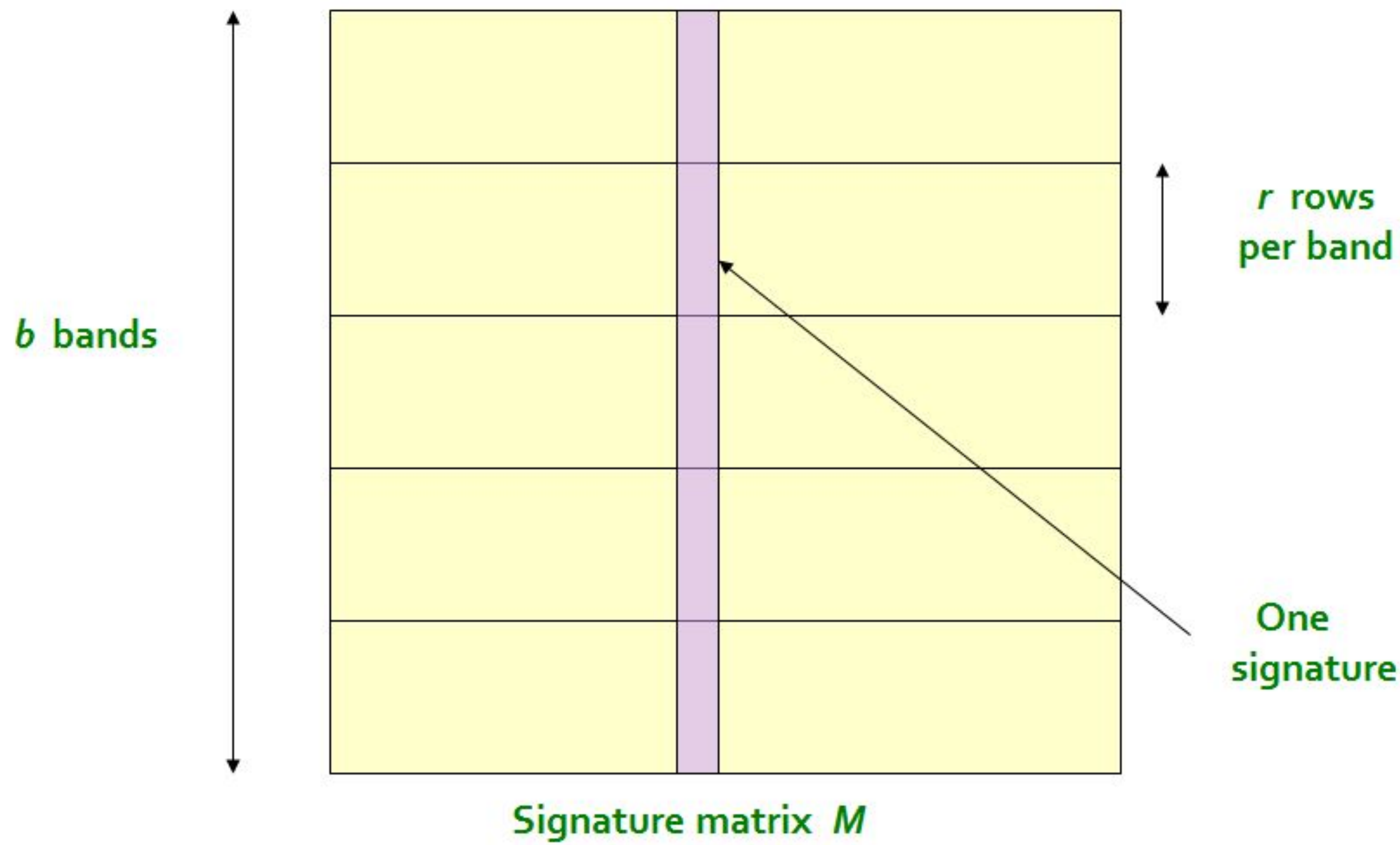
Approach: Hash multiple times over subsets of data: similar items are likely in the same bucket once.

Approach from MinHash: Hash columns of signature matrix

⇒ Candidate pairs end up in the same bucket.

Locality-Sensitive Hashing

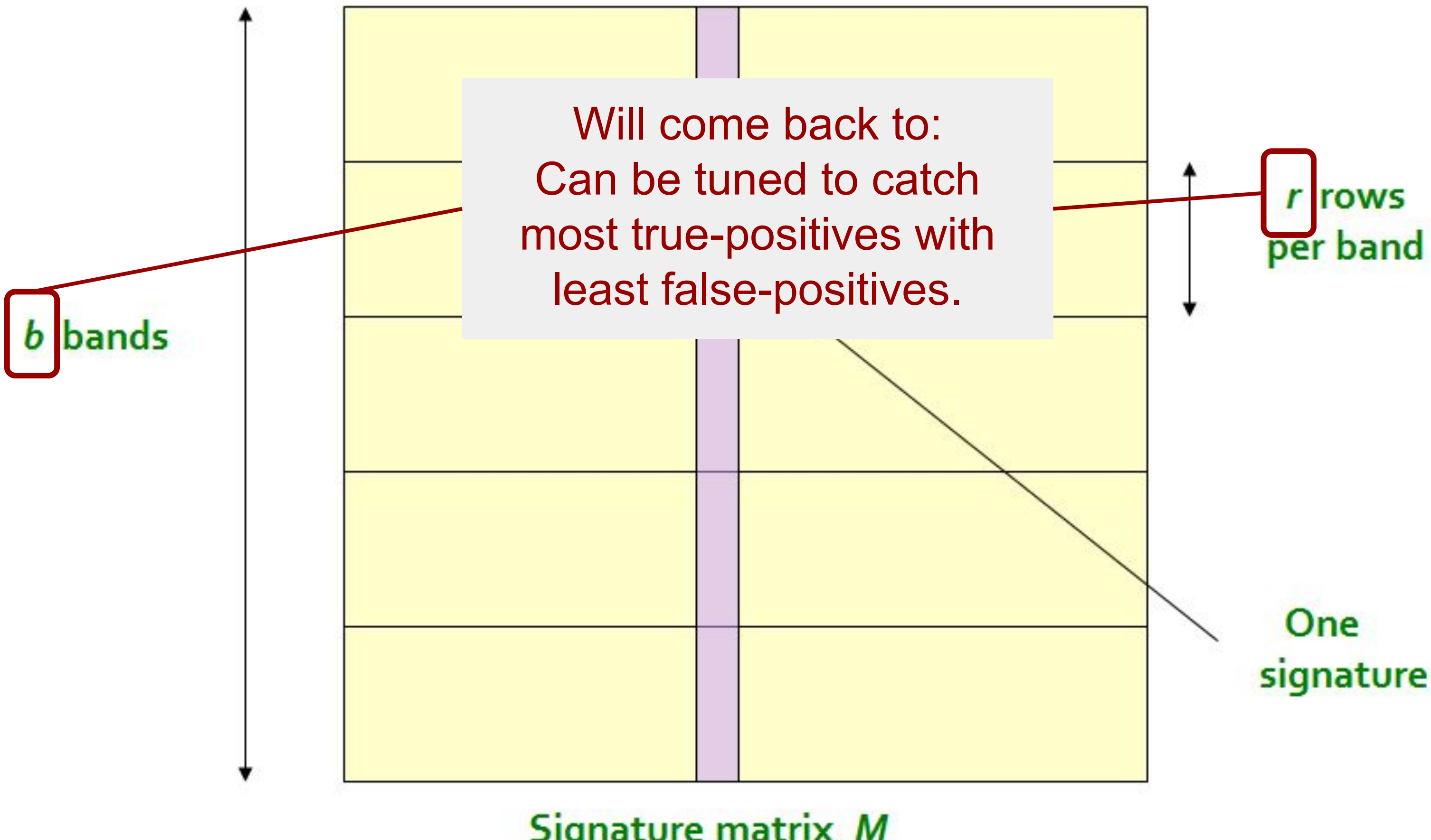
Step 1: Divide signature matrix into *b* bands



(Leskovec at al., 2014; http://www.mmds.org/)

Locality-Sensitive Hashing

Step 1: Divide into *b* bands

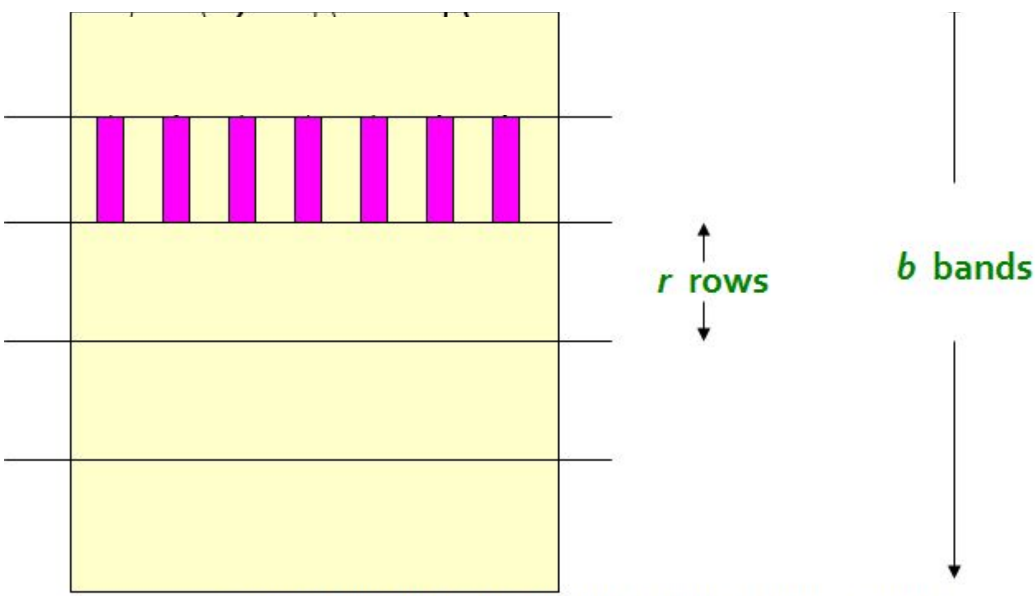


Will come back to:
Can be tuned to catch most true-positives with least false-positives.

(Leskovec at al., 2014; http://www.mmds.org/)

Locality-Sensitive Hashing

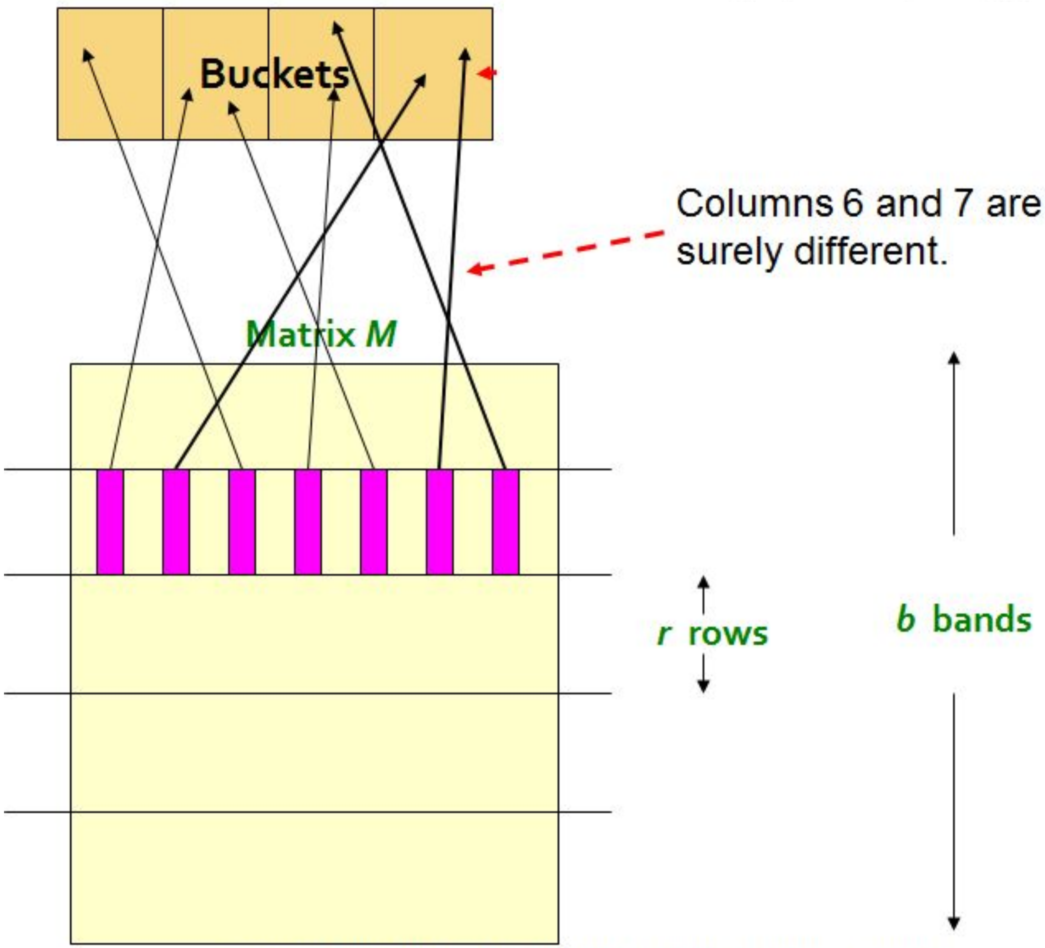
Step 1: Divide into *b* bands
Step 2: Hash columns within bands (one hash per band)



(Leskovec at al., 2014; http://www.mmds.org/)

Locality-Sensitive Hashing

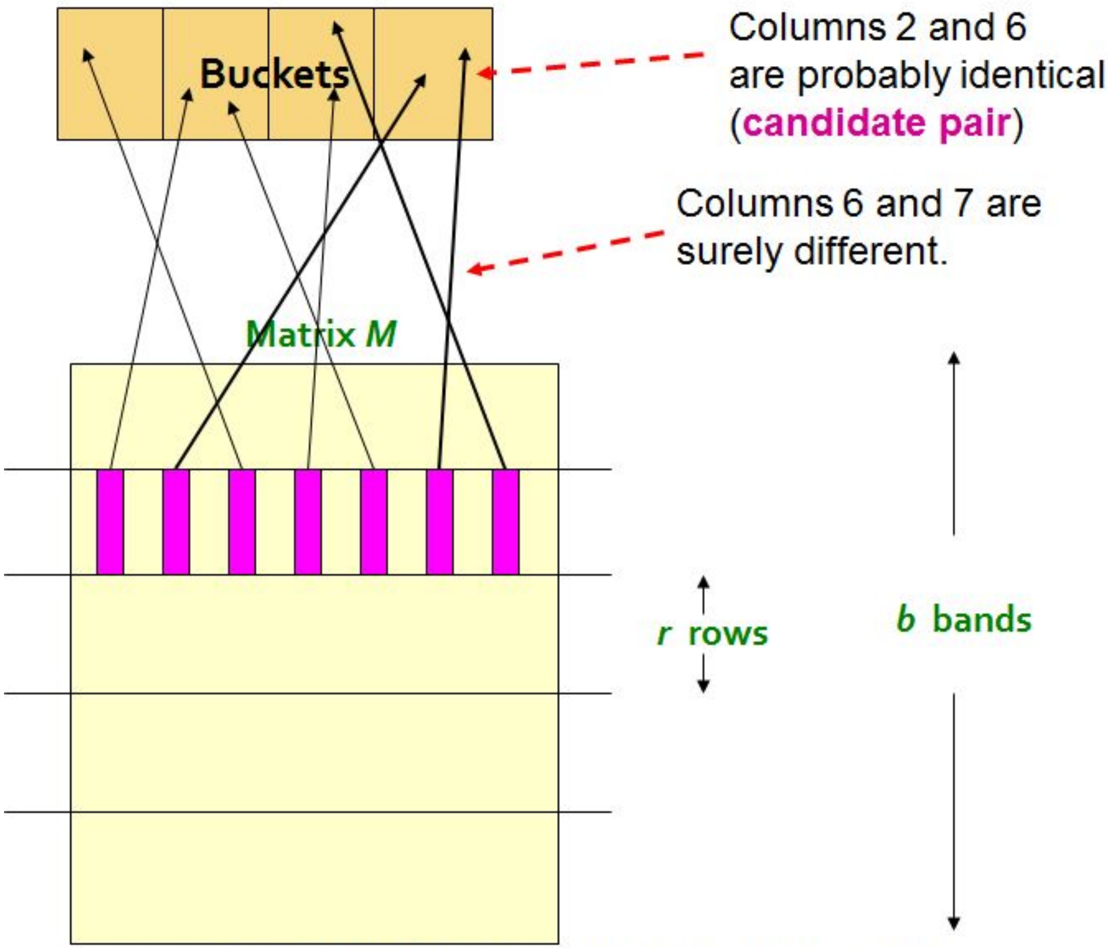
Step 1: Divide into *b* bands
Step 2: Hash columns within bands (one hash per band)



(Leskovec at al., 2014; http://www.mmds.org/)

Locality-Sensitive Hashing

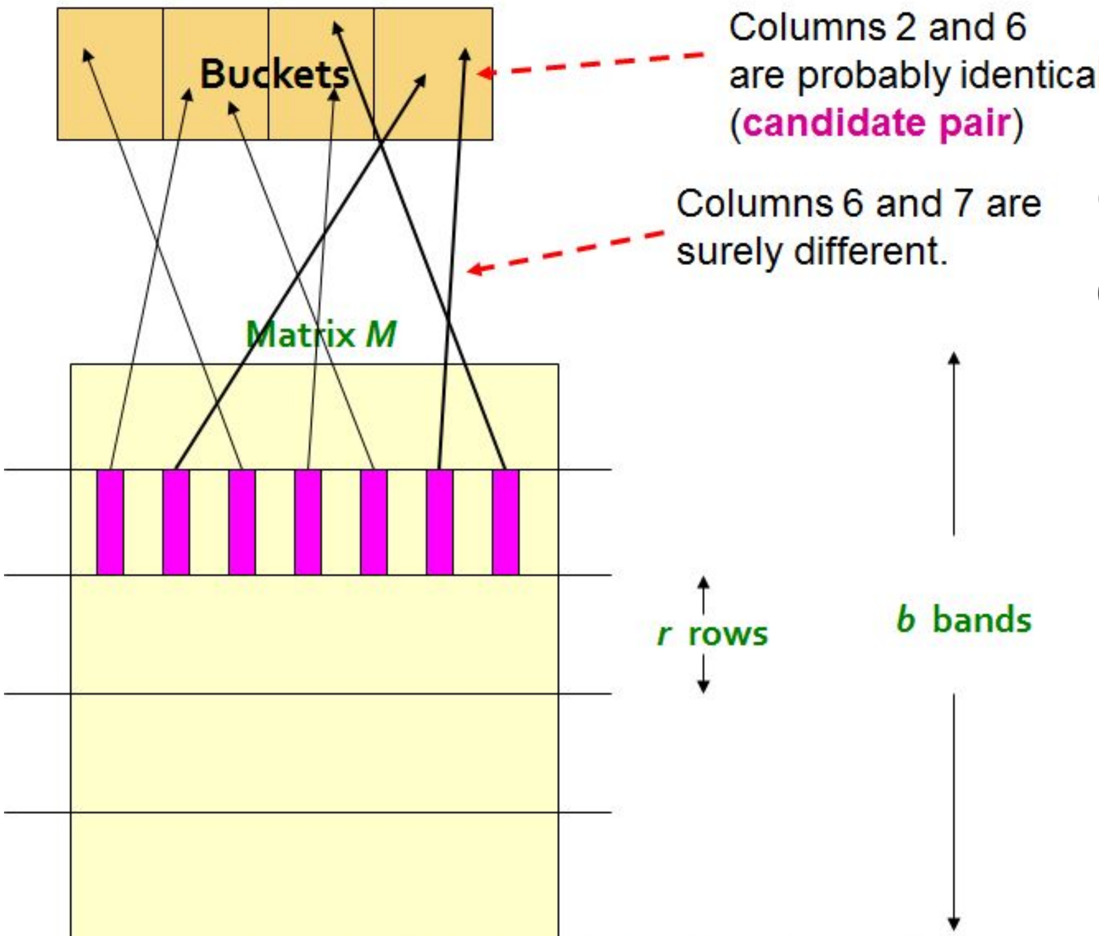
Step 1: Divide into *b* bands
Step 2: Hash columns within bands (one hash per band)



(Leskovec at al., 2014; http://www.mmds.org/)

Locality-Sensitive Hashing

Step 1: Divide into *b* bands
Step 2: Hash columns within bands (one hash per band)



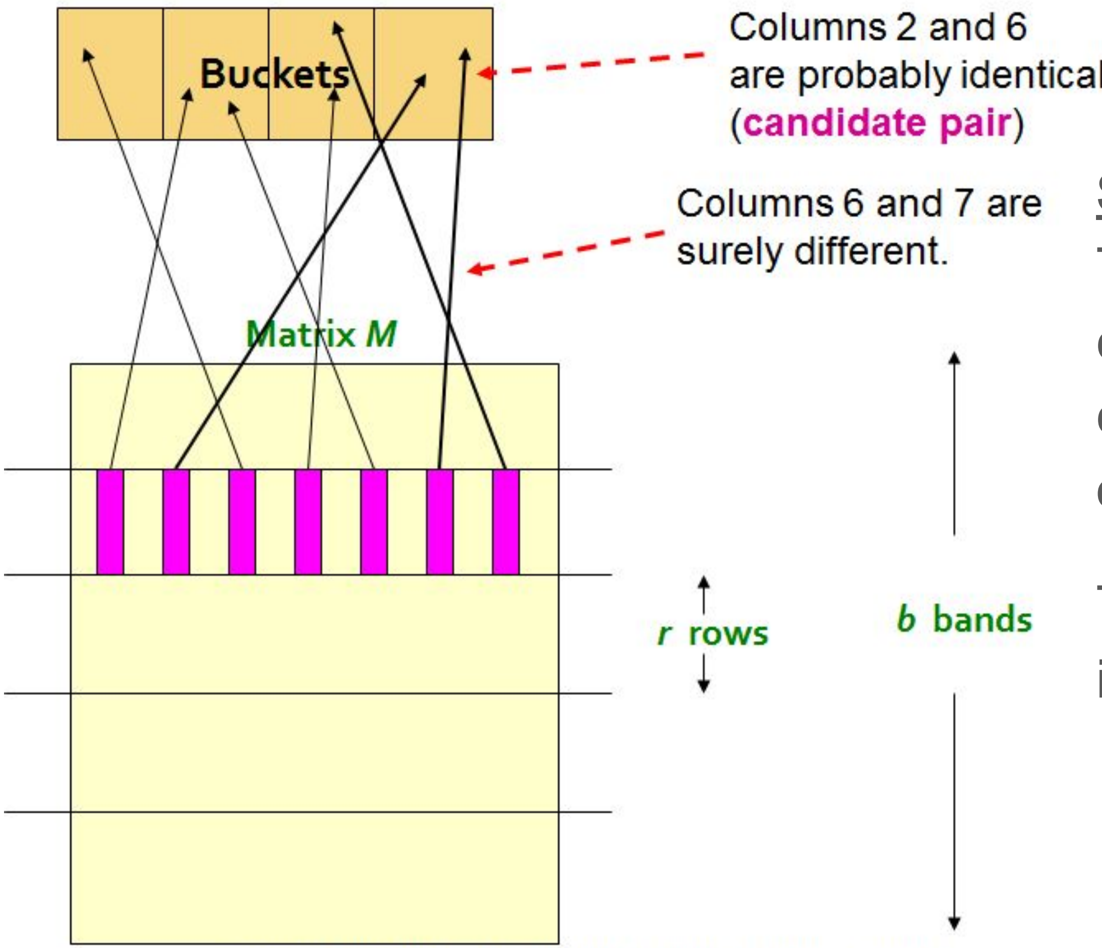
Criteria for being candidate pair:

- They end up in same bucket for at least 1 band.

(Leskovec at al., 2014; http://www.mmds.org/)

Locality-Sensitive Hashing

Step 1: Divide into *b* bands
Step 2: Hash columns within bands (one hash per band)

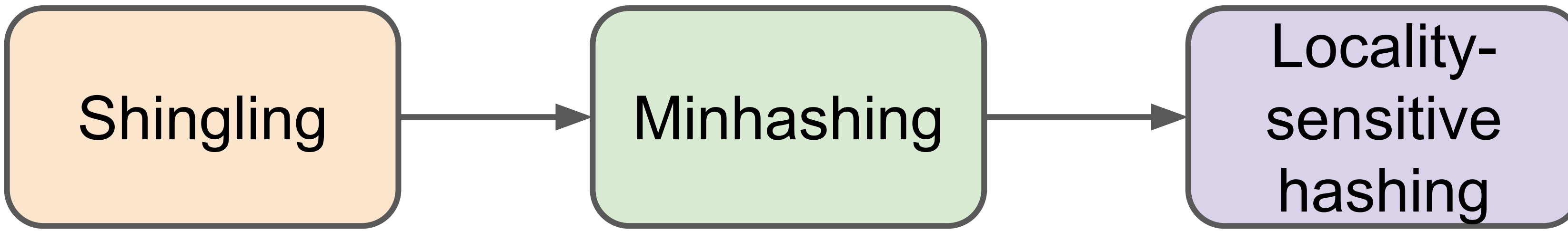


Simplification:
There are enough buckets compared to rows per band that columns must be identical in order to hash into same bucket.

Thus, we only need to check if identical within a band.

(Leskovec at al., 2014; http://www.mmds.org/)

Document-Similarity Pipeline



Probability of Agreement

- 100,000 documents
- 100 random permutations/hash functions/rows

  => if 4byte integers then 40Mb to hold signature matrix

  => still 100k choose 2 is a lot (~5billion)

Probability of Agreement

- 100,000 documents
- 100 random permutations/hash functions/rows

  => if 4byte integers then 40Mb to hold signature matrix

  => still 100k choose 2 is a lot (~5billion)
- 20 bands of 5 rows
- Want 80% Jaccard Similarity ; for any row $p(S_1 == S_2) = .8$

Probability of Agreement

- 100,000 documents
- 100 random permutations/hash functions/rows

  => if 4byte integers then 40Mb to hold signature matrix

  => still 100k choose 2 is a lot (~5billion)
- 20 bands of 5 rows
- Want 80% Jaccard Similarity ; for any row $p(S_1 == S_2) = .8$

$P(S_1==S_2 \mid b^{(5)})$: probability S1 and S2 agree within a given band

Probability of Agreement

- 100,000 documents
- 100 random permutations/hash functions/rows

  => if 4byte integers then 40Mb to hold signature matrix

  => still 100k choose 2 is a lot (~5billion)
- 20 bands of 5 rows
- Want 80% Jaccard Similarity ; for any row $p(S_1 == S_2) = .8$

$P(S_1 == S_2 \mid b^{(5)})$: probability S1 and S2 agree within a given band

$= 0.8^5 = .328$

(Leskovec at al., 2014; http://www.mmds.org/)

Probability of Agreement

- 100,000 documents
- 100 random permutations/hash functions/rows
  => if 4byte integers then 40Mb to hold signature matrix
  => still 100k choose 2 is a lot (~5billion)
- 20 bands of 5 rows
- Want 80% Jaccard Similarity ; for any row $p(S_1 == S_2) = .8$

$P(S_1{==}S_2 \mid b^{(5)})$: probability S1 and S2 agree within a given band

$= 0.8^5 = .328 \quad \Rightarrow \quad P(S_1 != S_2 \mid b) = 1 - .328 = .672$

(Leskovec at al., 2014; http://www.mmds.org/)

Probability of Agreement

- 100,000 documents
- 100 random permutations/hash functions/rows

  => if 4byte integers then 40Mb to hold signature matrix

  => still 100k choose 2 is a lot (~5billion)
- 20 bands of 5 rows
- Want 80% Jaccard Similarity ; for any row $p(S_1 == S_2) = .8$

$P(S_1{=}{=}S_2 \mid b^{(5)})$: probability S1 and S2 agree within a given band

$= 0.8^5 = .328 \quad \Rightarrow \quad P(S_1 != S_2 \mid b) = 1-.328 = .672$

$P(S_1 != S_2)$: probability S1 and S2 do not agree in any band

(Leskovec at al., 2014; http://www.mmds.org/)

Probability of Agreement

- 100,000 documents
- 100 random permutations/hash functions/rows

  => if 4byte integers then 40Mb to hold signature matrix

  => still 100k choose 2 is a lot (~5billion)
- 20 bands of 5 rows
- Want 80% Jaccard Similarity ; for any row $p(S_1 == S_2) = .8$

$P(S_1 == S_2 \mid b^{(5)})$: probability S1 and S2 agree within a given band

$= 0.8^5 = .328 \quad \Rightarrow \quad P(S_1 != S_2 \mid b) = 1 - .328 = .672$

$P(S_1 != S_2)$: probability S1 and S2 do not agree in any band

$= .672^{20} = .00035$

(Leskovec at al., 2014; http://www.mmds.org/)

Probability of Agreement

- 100,000 documents
- 100 random permutations/hash functions/rows

  => if 4byte integers then 40Mb to hold signature matrix

  => still 100k choose 2 is a lot (~5billion)
- 20 bands of 5 rows
- Want 80% Jaccard Similarity ; for any row $p(S_1 == S_2) = .8$

$P(S_1==S_2 \mid b)$: probability S1 and S2 agree within a given band

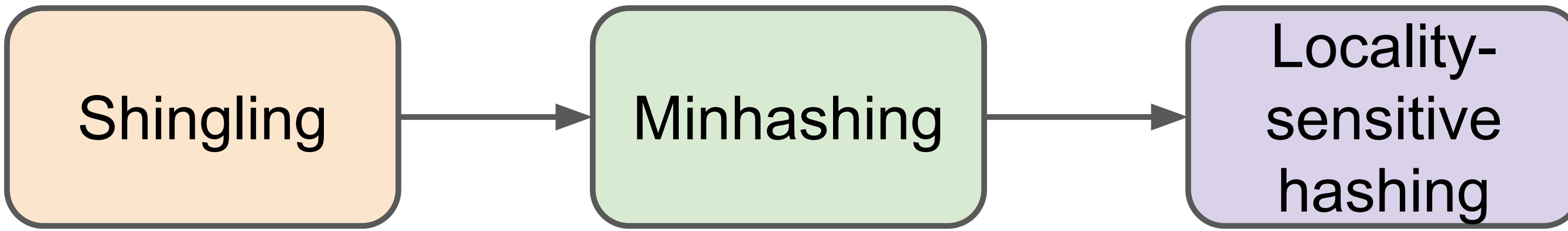
$= 0.8^5 = .328 \quad => \quad P(S_1 != S_2 \mid b) = 1-.328 = .672$

$P(S_1 != S_2)$: probability S1 and S2 do not agree in any band

$=.672^{20} = .00035$

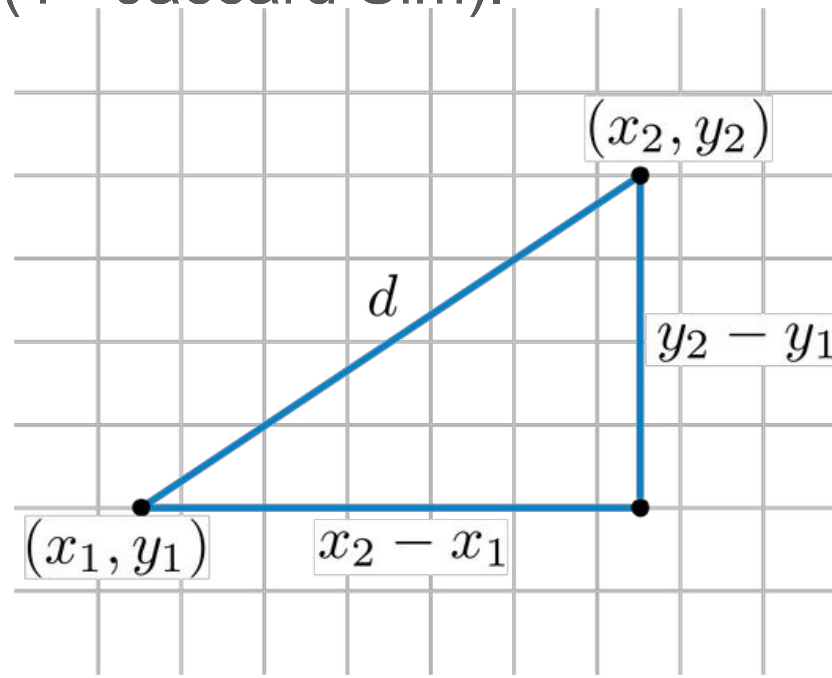
What if wanting 40% Jaccard Similarity?

Similarity Search



Distance Metrics

Pipeline gives us a way to find *near-neighbors* in *high-dimensional space* based on Jaccard Distance (1 - Jaccard Sim).

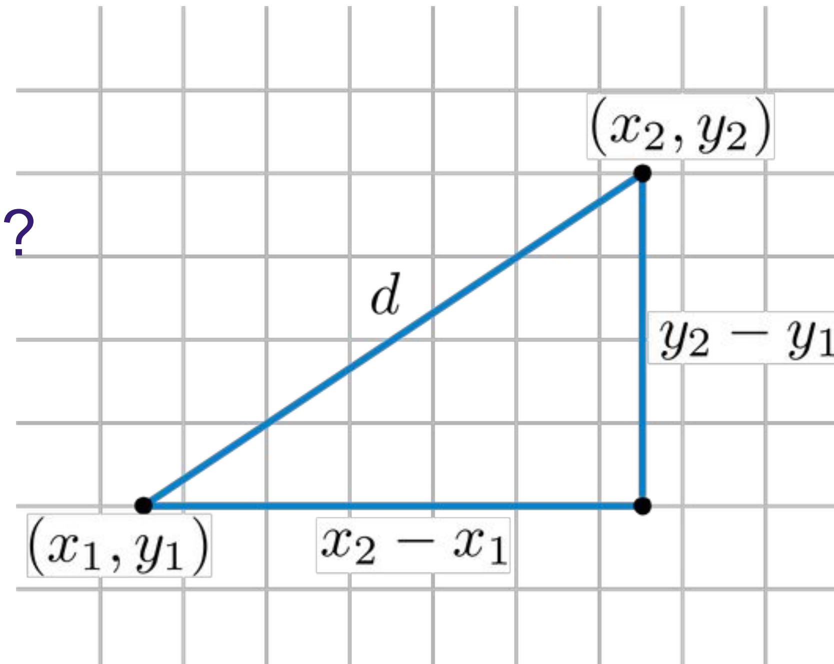


(http://rosalind.info/glossary/euclidean-distance/)

Distance Metrics

Pipeline gives us a way to find *near-neighbors* in *high-dimensional space* based on Jaccard Distance (1 - Jaccard Sim).

Typical properties of a distance metric, *d(point1,point2)*?



(http://rosalind.info/glossary/euclidean-distance/)

Distance Metrics

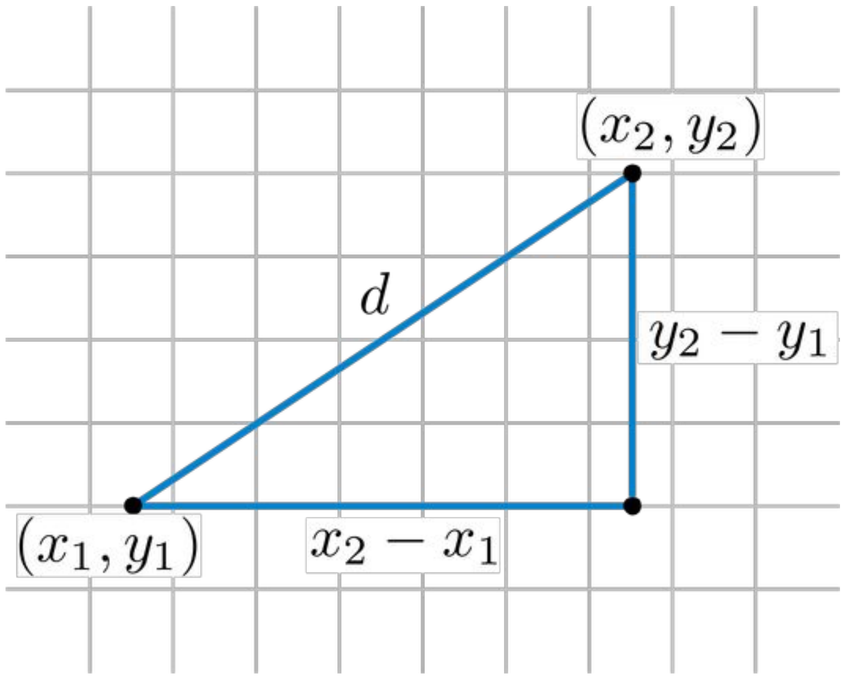
Pipeline gives us a way to find *near-neighbors* in *high-dimensional space* based on Jaccard Distance (1 - Jaccard Sim).

Typical properties of a distance metric, *d*:

d(a, a) = 0

d(a, b) = d(b, a)

d(a, b) ≤ d(a,c) + d(c,b)



(http://rosalind.info/glossary/euclidean-distance/)

Distance Metrics

Pipeline gives us a way to find *near-neighbors* in *high-dimensional space* based on Jaccard Distance (1 - Jaccard Sim).

There are other metrics of similarity. e.g:

- Euclidean Distance
- Cosine Distance

  …
- Edit Distance
- Hamming Distance

Distance Metrics

Pipeline gives us a way to find *near-neighbors* in *high-dimensional space* based on Jaccard Distance (1 - Jaccard Sim).

There are other metrics of similarity. e.g:

- Euclidean Distance

$$distance(X,Y) = \sqrt{\sum_{i}^{n}(x_i - y_i)^2}$$ ("L2 Norm")

- Cosine Distance

  …

- Edit Distance
- Hamming Distance

Distance Metrics

Pipeline gives us a way to find *near-neighbors* in *high-dimensional space* based on Jaccard Distance (1 - Jaccard Sim).

There are other metrics of similarity. e.g:

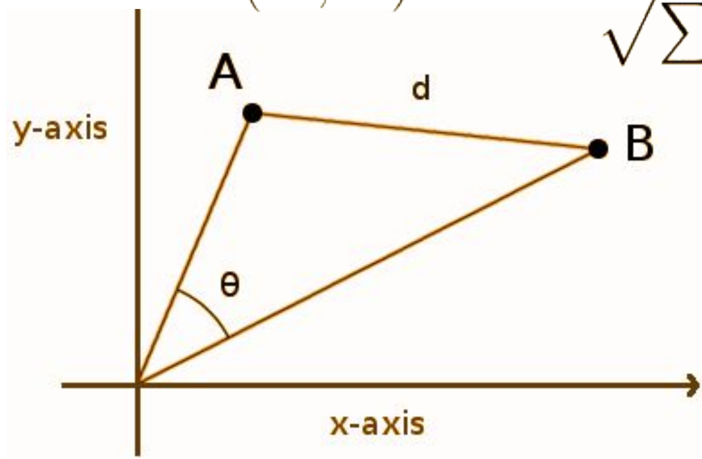
- Euclidean Distance
- Cosine Distance

  …

- Edit Distance
- Hamming Distance

$$distance(X, Y) = \sqrt{\sum_{i}^{n} (x_i - y_i)^2}$$ ("L2 Norm")

$$distance(X, Y) = 1 - \frac{\sum x_i y_i}{\sqrt{\sum x_i^2} \sqrt{\sum y_i^2}}$$



Distance Metrics

Locality Sensitive Hashing - Theory

LSH Can be generalized to many distance metrics by converting output to a probability and providing a lower bound on probability of being similar.

E.g. for euclidean distance:

- Choose random lines (analogous to hash functions in minhashing)
- Project the two points onto each line; match if two points within an interval